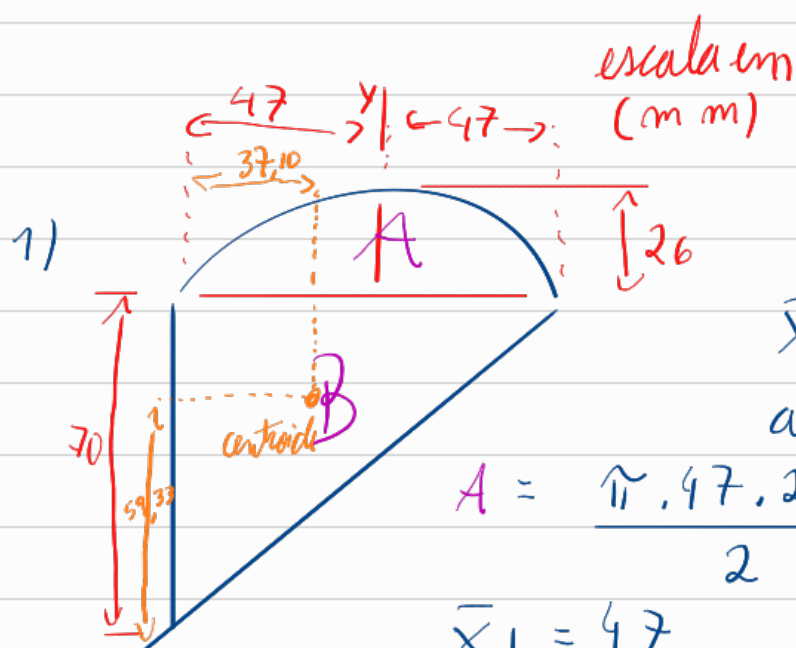




Semi-ellipse

$\bar{x}$ e $\bar{y}$ centroide fica mais abaixo e à esquerda

$$A = \frac{\pi \cdot 47 \cdot 26}{2} = 1919,51 \text{ mm}$$

$$\bar{x}_1 = 47$$

$$\bar{y}_1 = 70 + \frac{4 \cdot 26}{3\pi} = 81,03$$

$$\bar{y}_2 = 70 - \frac{70}{3}$$

$$B = \frac{47 \cdot 70}{2} = 3290 \quad \bar{x}_2 = \frac{47}{3} = 31,33 \quad \bar{y}_2 = 46,67$$

Centroide $(\bar{x}, \bar{y})$ Área Total = $1919,51 + 3290 = 5209,51$

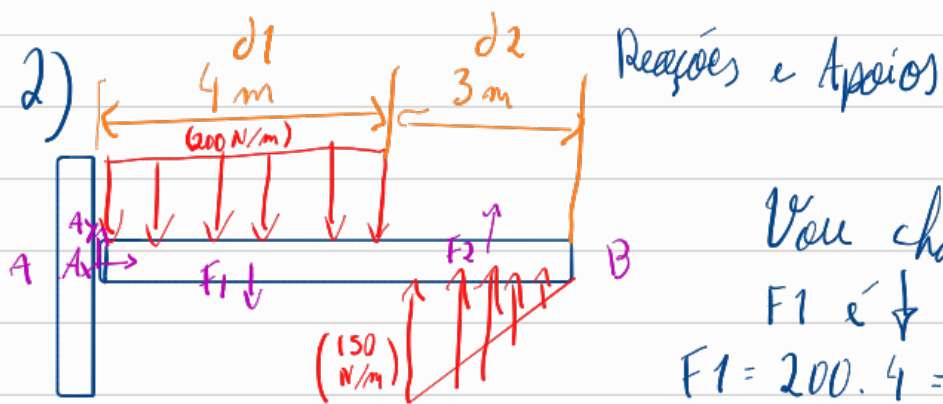
$$\bar{x} = \frac{(A \cdot \bar{x}_1) + (B \cdot \bar{x}_2)}{\text{Área Total}} = \frac{(1919,51 \cdot 47) + (3290 \cdot 31,33)}{5209,51}$$

$$\bar{x} = 37,10 \text{ mm}$$

$$\bar{y} = \frac{(A \cdot \bar{y}_1) + (B \cdot \bar{y}_2)}{\text{Área Total}} = \frac{(1919,51 \cdot 81,03) + (3290 \cdot 46,67)}{5209,51}$$

$$\bar{y} = 59,33 \text{ mm}$$

$$(37,10; 59,33) \text{ mm}$$



Vou chamar de F_1 e F_2 !
 F_1 é $\downarrow$ porém F_2 é triangular!
 $F_1 = 200 \cdot 4 = 800 \text{ N} \downarrow$ $F_2 = \frac{150 \cdot 3}{2} = 225 \text{ N} \uparrow$

Sei que F_1 atua na metade de d_1 , logo:

$$d_1 = 4, \quad \frac{d_1}{2} = 2 \text{ m de A } (x_1)$$

Já F_2 , o centroide está a $\frac{1}{3}$ de d_2 , logo:

$$d_2 = 3, \quad \frac{1}{3} \cdot 3 = 1, \quad 1 + d_1 = 5 \text{ m de A } (x_2)$$

(Se for triangular, lembrar de $\frac{1}{3}$)

$$\rightarrow \sum F_x = 0$$

$$A_x = 0$$

$$+\uparrow \sum F_y = 0$$

$$A_y - F_1 + F_2 = 0$$

$$A_y - 800 + 225 = 0$$

$$A_y = 575 \text{ N} \uparrow$$

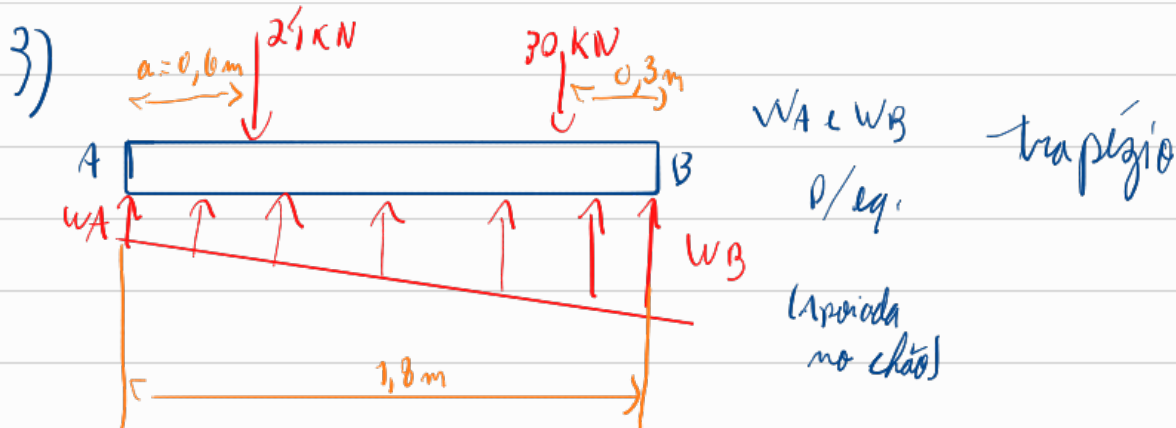
$$\sum M_A = 0$$

$$M_A - (F_1 \cdot x_1) + (F_2 \cdot x_2) = 0$$

$$M_A - (800 \cdot 2) + (225 \cdot 5) = 0$$

$$M_A - (1600) + (1125) = 0$$

$$\hookrightarrow M_A = 475 \text{ N} \cdot \text{m}$$



$$F_A = \frac{w_A \cdot 1,8}{2} = 0,9 \cdot w_A \quad x_A = \frac{1,8}{3} = 0,6 \text{ m a partir de A}$$

$$F_B = \frac{w_B \cdot 1,8}{2} = 0,9 w_B \quad x_B = 2 \cdot \left(\frac{1,8}{3}\right) = 1,2 \text{ m}$$

(24 kN em 0,6 m e 30 kN em 1,8 - 0,3 = 1,5 m)

$$\sum M_A = 0$$

$$(F_A \cdot x_A) + (F_B \cdot x_B) - (24 \cdot 0,6) - (30 \cdot 1,5) = 0$$

$$(0,9 w_A \cdot 0,6) + (0,9 w_B \cdot 1,2) - 14,4 - 45 = 0$$

$$0,54 w_A + 1,08 w_B = 59,4$$

$$\sum \bar{F}_y = 0$$

$$F_A + F_B - 24 - 30 = 0$$

$$0,9 w_A + 0,9 w_B = 54$$

$$w_A + w_B = \frac{54}{0,9}$$

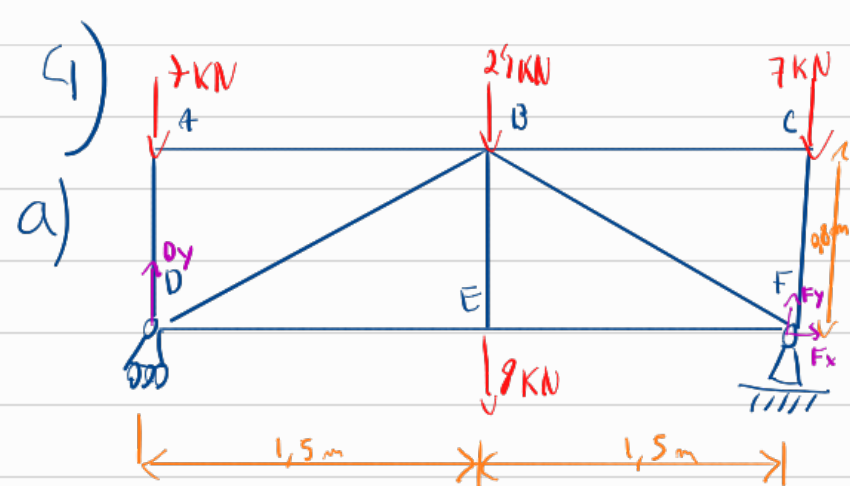
$$w_A + w_B = 60$$

$$w_A = 60 - w_B$$

$$0,54 \cdot (60 - w_B) + 1,08 w_B = 59,4$$

$$w_B = 50 \text{ kN/m}$$

$$w_A = 10 \text{ kN/m}$$



é simétrico!

Reações de Apoio

$$7 + 24 + 7 + 2 = 46 \text{ kN}$$

$$D_y = F_y = \frac{46}{2} = 23 \text{ kN} \uparrow$$

$$F_x = 0$$

$$BD, BF = L = \sqrt{1,5^2 + 0,8^2} = 1,7 \text{ m}$$

$$\sin \theta = \frac{0,8}{1,7} = \frac{8}{17} = 28,07^\circ$$

$$\cos \theta = \frac{1,5}{1,7} = \frac{15}{17} = 28,07^\circ$$

No A:

$$F_{AD} = 7 \text{ kN (C)}$$

$$F_{AB} = 0$$

No D:

No E:

$$F_{BE} = 8 \text{ kN (T)}$$

$$\sum F_y = 23 - 7 - F_{BD} \cdot \sin(\theta) = 0$$

$$16 - F_{BD} \cdot \left(\frac{8}{17}\right) = 0$$

$$F_{BD} = 34 \text{ kN (C)}$$

$$\sum F_x = F_{DE} - F_{BD} \cdot \cos(\theta) = 0$$

$$F_{DE} - 34 \cdot \left(\frac{15}{17}\right) = 0$$

$$F_{DE} = 30 \text{ kN (T)}$$

$$F_{CF} = F_{AD} = 7 \text{ kN (C)}$$

$$F_{BC} = F_{AB} = 0$$

$$F_{BF} = F_{BD} = 34 \text{ kN (C)}$$

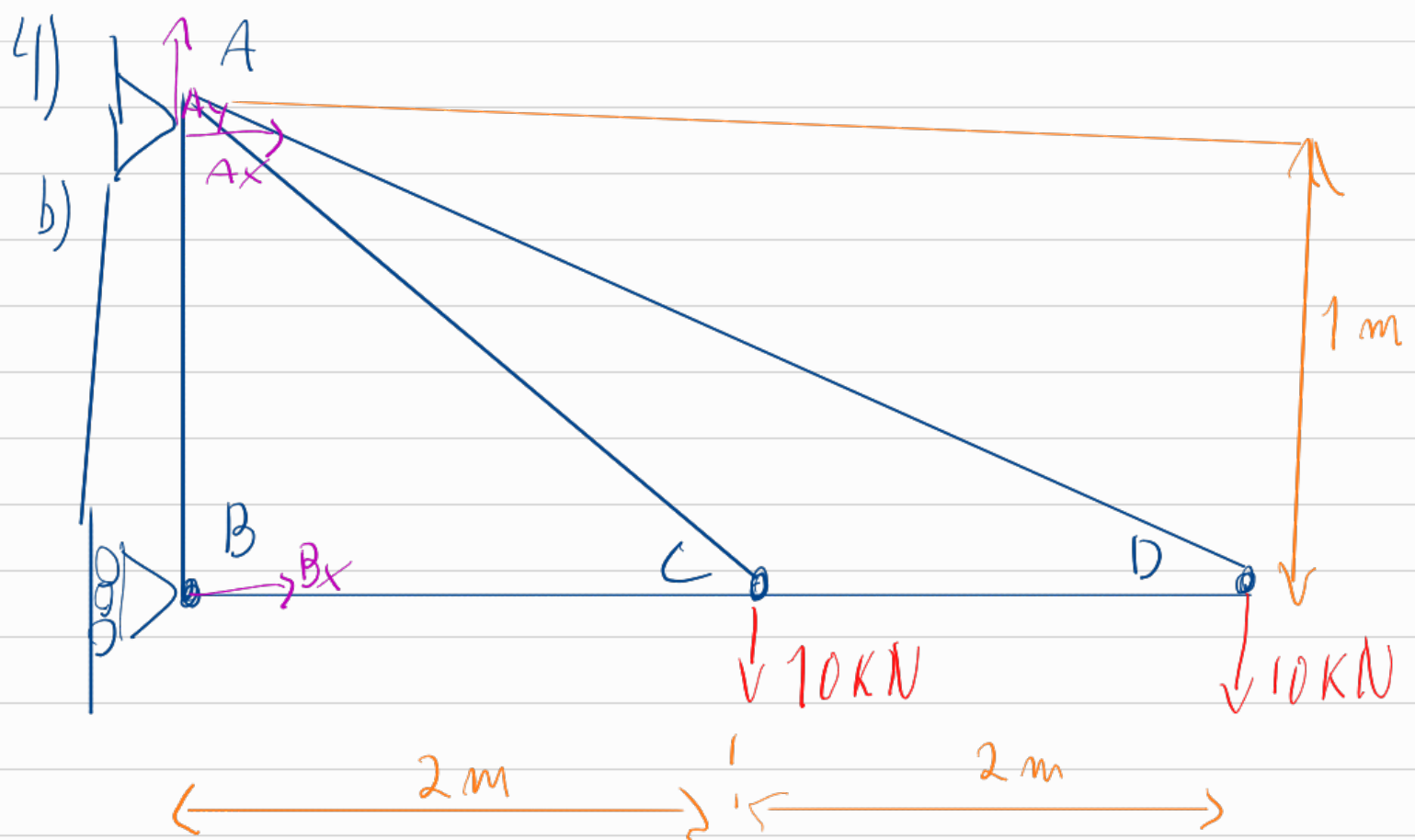
$$F_{EF} = F_{DE} = 30 \text{ kN (T)}$$

$$F_{CF} = F_{AD} = 7 \text{ kN (C)}$$

$$F_{BC} = F_{AB} = 0$$

$$F_{BF} = F_{BD} = 34 \text{ kN (C)}$$

$$F_{EF} = F_{DE} = 30 \text{ kN (T)}$$



$$\sum M_A = 0$$

$$B_x \cdot 1 - 10 \cdot 2 - 10 \cdot 4 = 0$$

$$B_x = 60 \text{ kN} \rightarrow$$

$$\text{No } B = F_{AB} = 0$$

$$F_{BC} = 60 \text{ kN (C)}$$

$$F_{BC} - F_{CD} - F_{AC} \cdot x =$$

$$60 - 10 - 10\sqrt{5} \cdot \left(\frac{2}{\sqrt{5}}\right) = 0$$

$$\text{No } D = \sqrt{1^2 + 4^2} = \sqrt{17}$$

$$\sum F_y = F_{AD} \cdot \left(\frac{1}{\sqrt{17}}\right) - 10 = 0$$

$$F_{AD} = 10\sqrt{17} \Rightarrow 41,23 \text{ kN (T)}$$

$$\sum F_x = F_{CD} - F_{AD} \cdot \left(\frac{4}{\sqrt{17}}\right) = 0$$

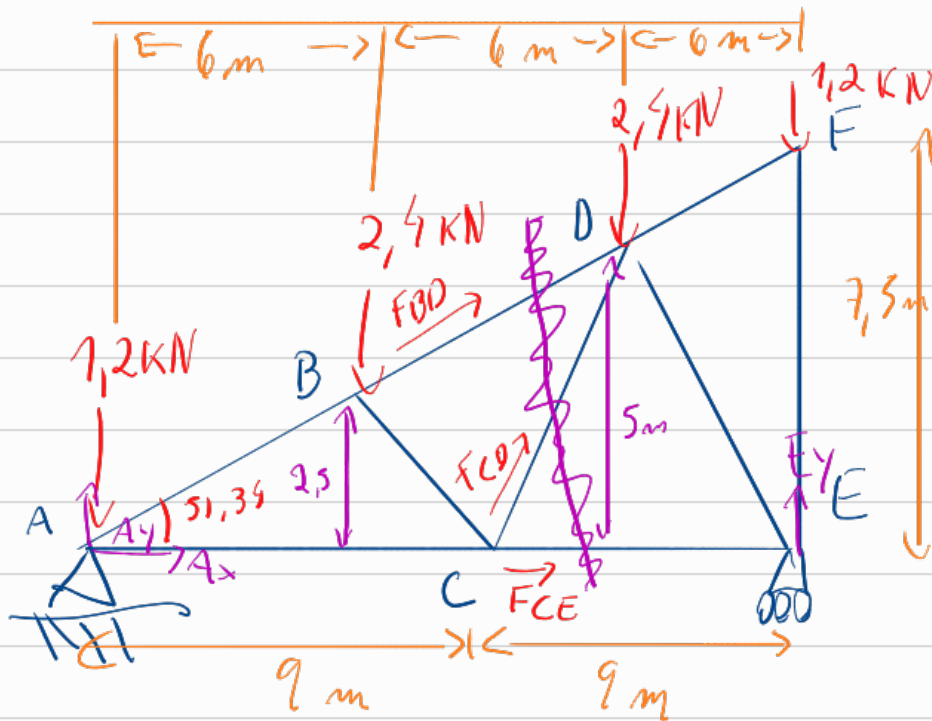
$$F_{CD} = 10\sqrt{17} \cdot \left(\frac{4}{\sqrt{17}}\right) = 40 \text{ kN (C)}$$

$$\text{No } C:$$

$$\sum F_y = F_{AC} \cdot \left(\frac{1}{\sqrt{5}}\right) - 10 = 0 \rightarrow F_{AC} = 10\sqrt{5}$$

$$\Rightarrow 22,36 \text{ kN (T)}$$

5) F BD, CD, CE Indique T ou C



$$\begin{aligned} \text{triângulo } h=18 \text{ m} \cdot 7.5 \\ (5-12 \cdot 13) \\ \sin \theta = \frac{7.5}{19.5} \\ \cos \theta = \frac{18}{19.5} \end{aligned}$$

$$\text{carga total} = 7.2 \text{ kN}$$

$$1.2 + 2.4 + 2.4 + 1.2$$

$$\frac{\text{base}}{\text{rampa}} = \frac{18}{19.5} \quad \arcsin\left(\frac{18}{19.5}\right) = 22.62^\circ$$

$$\frac{\text{altura}}{\text{rampa}} = \frac{7.5}{19.5} \quad \arcsin\left(\frac{7.5}{19.5}\right) = 67.38^\circ$$

$$\hookrightarrow M_A = 0$$

$$\begin{aligned} &-(2.4 \text{ kN} \cdot 6 \text{ m}) - (2.4 \text{ kN} \cdot 12 \text{ m}) \\ &-(1.2 \text{ kN} \cdot 18 \text{ m}) + (E_y \cdot 18 \text{ m}) = 0 \\ \Rightarrow E_y &= 3.6 \text{ kN} \uparrow \end{aligned}$$

o triângulo AFE é

$$A = (0, 0) \quad F = (18, 7.5) \quad E = (18, 0)$$

$$\begin{aligned} \sqrt{\text{base}^2 + \text{altura}^2} &= \text{hip} \\ \sqrt{18^2 + 7.5^2} &= 19.5 \end{aligned}$$

$$\hookrightarrow \sum F_y = 0$$

$$A_y + E_y - 1.2 - 2.4 - 2.4 - 1.2 = 0$$

$$A_y + 3.6 - 7.2 = 0$$

$$A_y = 3.6 \uparrow$$

FCE

$$\sum M_D = 0 \quad D_x = 12 \quad D_y = 5$$

$$-(A_y \cdot 12) + (1.2 \cdot 12) + (2.4 \cdot 6) + F_{CE} \cdot 5 = 0$$

$$F_{CE} = 2.88 \text{ (T)}$$

FBD

$$\sum M_C = 0 \quad B_x = 9 \quad B_y = 2.5$$

$$F_{BD_x} = F_{BD} \cdot \cos 22.62 =$$

$$F_{BD_y} = F_{BD} \cdot \sin 22.62 =$$

$$\frac{y_B}{6} = \frac{7.5}{18}$$

$$y_B = 2.5 \text{ m}$$

$$\frac{y_D}{12} = \frac{7.5}{18}$$

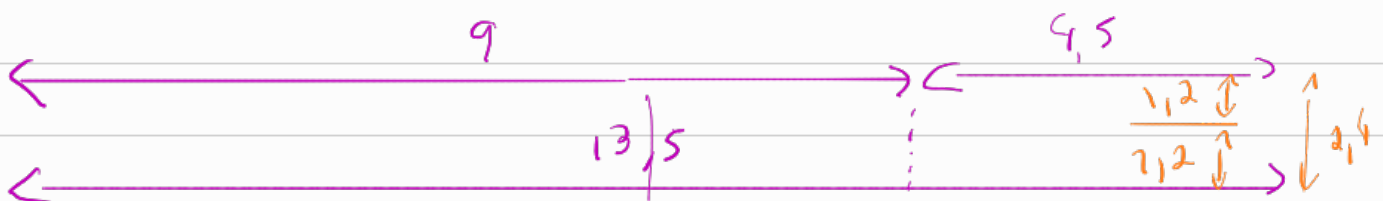
$$y_D = 5 \text{ m}$$

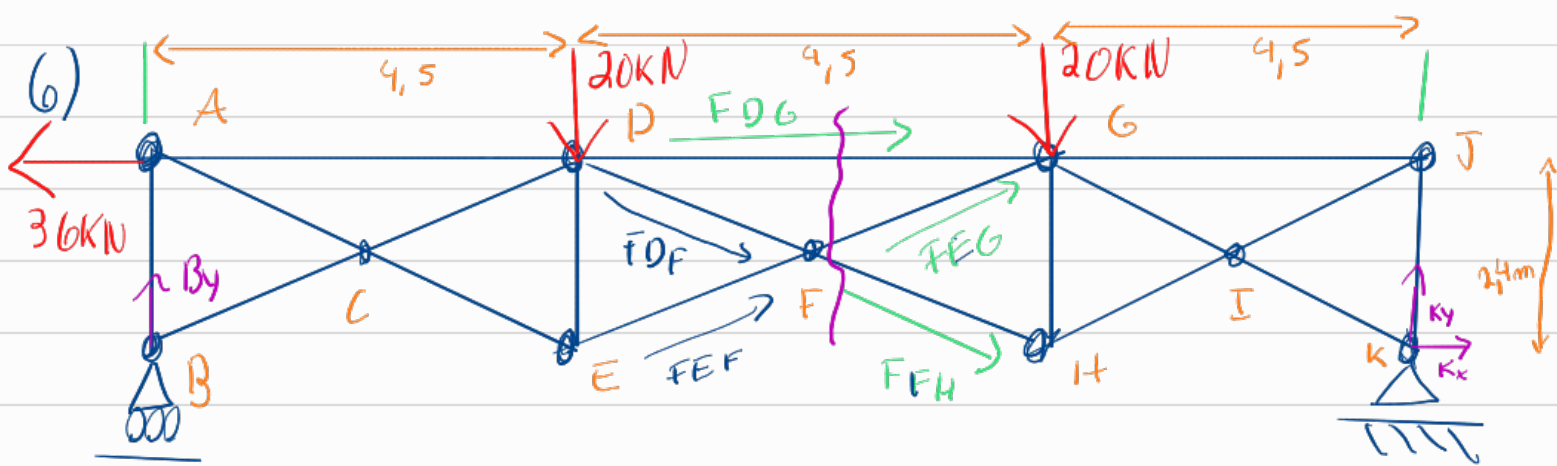
$$\begin{aligned}
 & - (3,6 \text{ kN} \cdot 9) + (1,2 \cdot 9) + (2,4 \cdot 3) - (F_{BD} \cdot \cos 22,62 \cdot 2,5) \\
 & - (F_{BD} \cdot \sin 22,62 \cdot 3) = 0 \\
 & \boxed{F_{BD} = -4,16 \text{ kN (C)}} \quad \text{or} \quad \frac{3}{5}
 \end{aligned}$$

$$\begin{aligned}
 & F_{CD} \\
 & \sum F_y = 0 \quad \rightarrow = -1,60 \text{ kN} \\
 & A_y - 1,2 - 2,4 + F_{BD_y} + F_{CD_y} = 0 \\
 & 3,6 - 1,2 - 2,4 + (-1,60) + F_{CD} (\sin 59,04) = 0 \\
 & \boxed{F_{CD} = 1,87 \text{ kN (T)}}
 \end{aligned}$$

$$\sum F_x = 0$$

$$\begin{aligned}
 & F_{CE} + F_{BD_x} + F_{CD_x} = 0 \\
 & 2,88 + (-4,86 \cdot 0,9231) + (1,87 \cdot 0,5145) = 0 \\
 & 2,88 - 3,89 + 0,96 = 0
 \end{aligned}$$





$$\rightarrow \sum F_x = 0$$

$$-36 + K_x = 0$$

$$K_x = 36 \text{ kN}$$

$$\uparrow \sum F_y = 0$$

$$-20 - 20 + B_y + K_y = 0$$

$$B_y + K_y = 40$$

$$B_y = 26,4 \text{ kN}$$

$$\sum M_B = 0$$

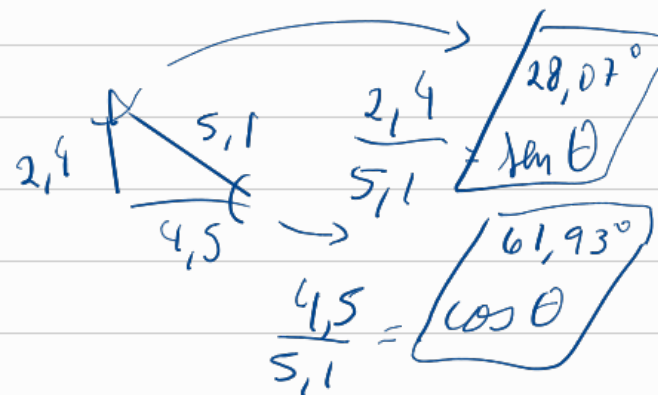
$$(36 \text{ kN} \cdot 2,4 \text{ m}) - (20 \text{ kN} \cdot 4,5 \text{ m}) - (20 \text{ kN} \cdot 9 \text{ m})$$

$$+ (K_y \cdot 13,5) = 0$$

$$K_y = 13,6 \text{ kN} \uparrow$$

comp diagonal $> \sqrt{4,5^2 + 2,4^2} = 5,1 \text{ m}$

no corta



$$\sum F_y = 0$$

$$13,6 - 20 + F_{DH} \cdot \sin \theta - F_{EG} \cdot \sin \theta = 0$$

$$-6,4 + F_{DH} \cdot \sin 28,07 - F_{EG} \cdot \sin 28,07 = 0$$

$$F_{DH} - F_{EG} = \frac{6,4}{\sin 28,07}$$

$$F_{DH} - F_{EG} = 13,6 \text{ kN}$$

$$F_{DH} = 13,6 \text{ kN (T)}$$

$$F_{FH} = \frac{1}{2} \cdot 13,6 = 6,8 \text{ kN (T)}$$

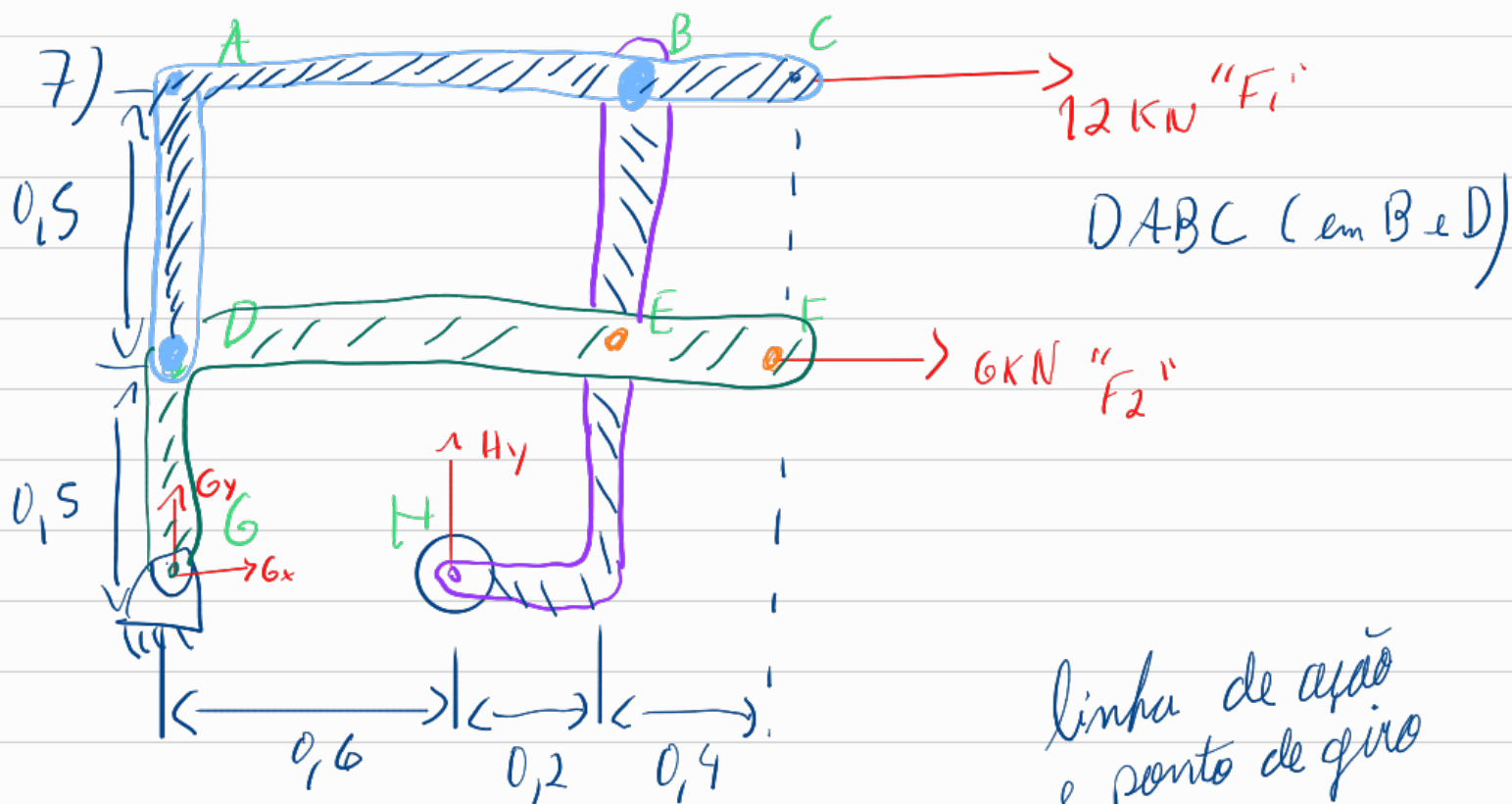
$$F_{EG} = 0$$

$$F_{FG} = 0$$

$$\sum M_H = 0$$

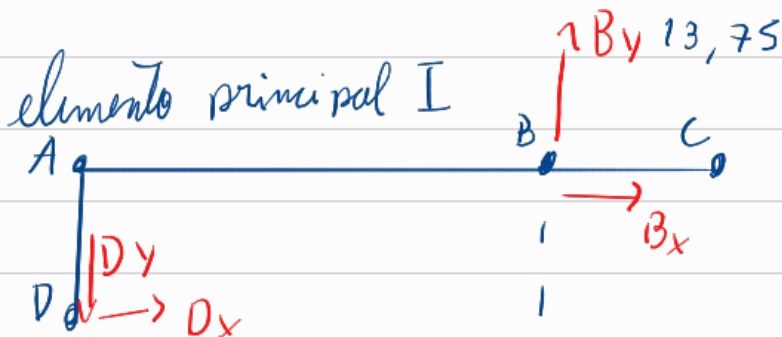
$$FDG \cdot 2,1 + 13,6 \cdot 4,5 = 0$$

$$FDG = -25,5 \text{ kN (C)}$$



linha de ação
e ponto de giro

F₁ e F₂ são horizontais
baixo, multiplico por y



B_x (invertido)

$$+\circlearrowleft \sum M_G = 0$$

$$H_y \cdot 0,6 - 12 \cdot 1 - 6 \cdot 0,5 = 0$$

$$H_y = 25 \text{ kN } \uparrow$$

$$+\uparrow \sum F_y = 0$$

$$G_y + 25 = 0; G_y = -25 \text{ kN } \downarrow$$

$$+\rightarrow \sum F_x = 0$$

$$G_x + 12 + 6 = 0; G_x = -18 \text{ kN } \leftarrow$$

bagunçado! 😊 assume se der!

$$+\circlearrowleft \sum M_D = 0$$

$$B_y \cdot 0,8 - 10 \cdot 0,5 - 12 \cdot 0,5 = 0$$

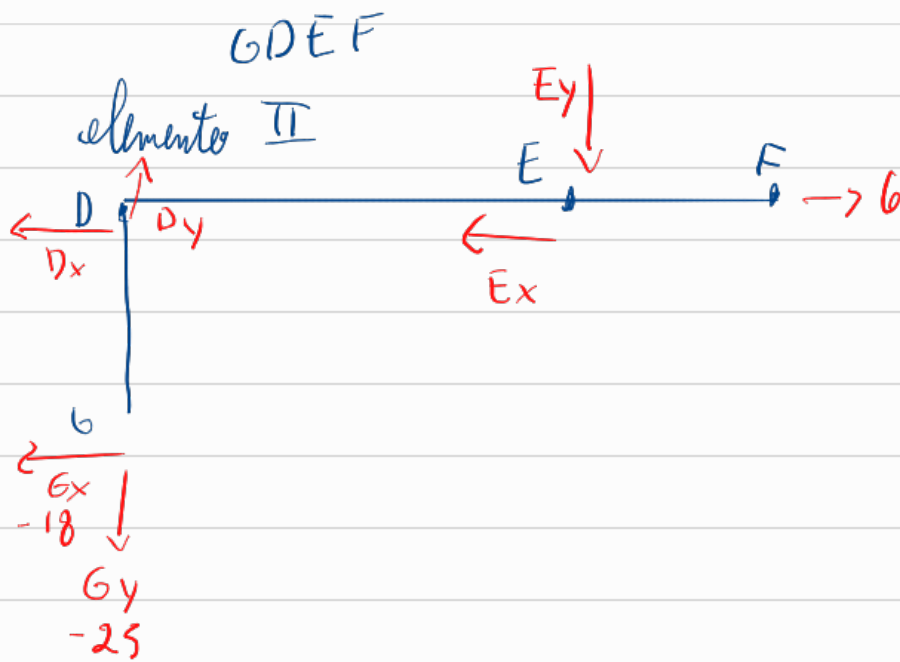
$$B_y = 13,75 \text{ kN } \uparrow$$

$$+\rightarrow \sum F_x = 0$$

kN

$$D_x = 10 + 12 = 0; D_x = -22 \text{ kN } \leftarrow$$

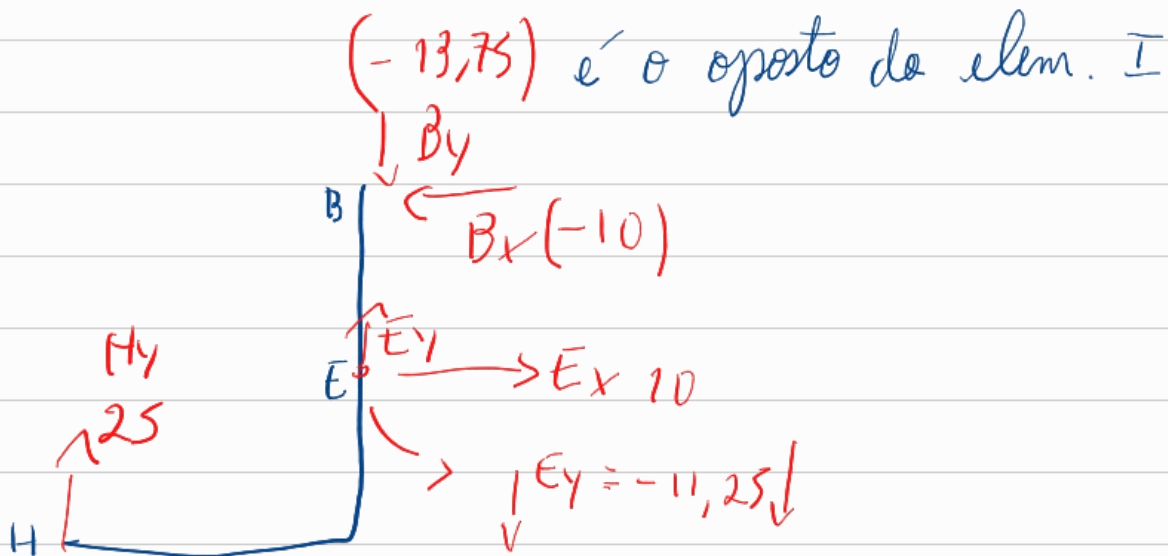
$$+\uparrow \sum F_y = 0; D_y + 13,75 = 0; D_y = -13,75 \text{ kN } \downarrow$$



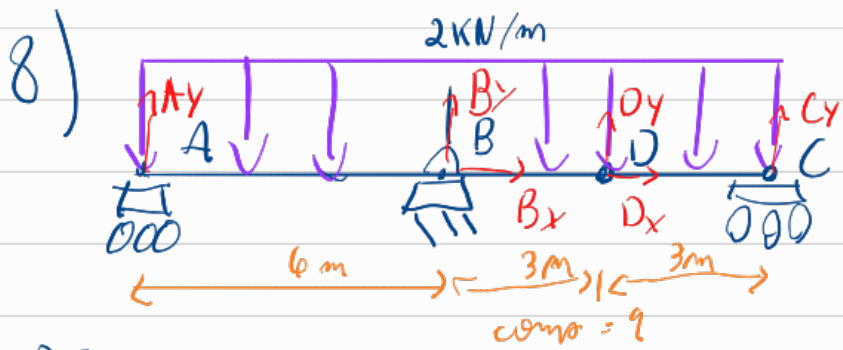
$$\begin{aligned} \sum F_y &= 0 \\ -25 + 13,75 - E_y &= 0 \\ E_y &= -11,25 \text{ kN} \uparrow \\ \sum F_x &= 0 \\ -18 + 22 + 6 - E_x &= 0 \\ E_x &= 10 \end{aligned}$$

elemento III - BEH

$$\begin{aligned} \sum M_E &= 0 \\ -25 \cdot 0,2 + B_x \cdot 0,5 &= 0 \\ B_x &= 10 \text{ kN} \leftarrow \end{aligned}$$



Ficou meio bagunçado esse, mas é compreensível, elementos atrás do pino, é só inverter o sinal } prestar atenção no giro



B, fixo, A, D e C, móveis

Dividir DC e resto

- DC

$$FDC = 2 \text{ kN} \cdot 3 \text{ m} = 6 \text{ kN} \text{ (atua no meio de DC)}$$

$$\sum M_D = 0$$

$$C_y \cdot 3 - 6 \cdot 1,5 = 0$$

$$C_y = 3 \text{ kN} \uparrow$$

$$\sum F_y = 0$$

$$D_y + C_y - 6 = 0; D_y = 3 \text{ kN} \uparrow \text{ [AD tem que fazer 3 kN} \uparrow \text{ p/ sustentar DC]}$$

- ABD

$$F_{AB} = \begin{matrix} \text{(kN/m)} \\ \downarrow 2 \cdot 6 = 12 \text{ kN (no meio entre A e B)} \downarrow \\ \downarrow 3 \text{ kN} \downarrow \end{matrix}$$

$$F_{AD} = 2 \cdot 3 = 6 \text{ kN} \text{ (no meio entre B e D)} \downarrow$$

$$\sum M_B = 0$$

$$-A_y \cdot (6) + 12 \cdot (3) - 6 \cdot (1,5) - 3 \cdot (3) = 0$$

$$A_y = 3 \text{ kN} \uparrow$$

$$\sum F_x = 0$$

$$A_y + B_y - F_{AB} - F_{BD} - D_y = 0$$

$$B_y = 18 \text{ kN} \uparrow$$

$$\sum F_y = 0$$

$$B_x = 0$$

$$9) F_{em} \quad C, D \text{ e } E$$

$$T = 500 \text{ N}$$

em D

$$D_x = 500 \text{ N}$$

$$D_y = 0$$

$$F_D = 500 \text{ N}$$

em C

$$C_x = 500 \text{ N}$$

$$C_y = 500 \text{ N}$$

$$F_C = \sqrt{C_x^2 + C_y^2}$$

$$F_C = 707,1 \text{ N}$$

$$\text{em E} \quad \text{---} + \sum M_A = 0$$

$$B_y \cdot 1 - 500 \cdot 2 = 0$$

$$B_y = 1000 \text{ N}$$

- BD

$$\sum F_y = 0$$

$$B_y + E_y = 0$$

$$1000 + E_y = 0$$

$$E_y = -1000 \text{ N}$$

$$\sum M_B = 0 \quad (D_x = 500)$$

$$500 \cdot 2 + |E_y| \cdot 0,5 + E_x \cdot 0,5 = 0$$

$$E_x = -3000 \text{ N}$$

$$F_E = \sqrt{E_x^2 + E_y^2}$$

$$F_E = 3162,3 \text{ N}$$

